Calc BC  9-1 Pg 481  # 21-25, 27-28, 31

(21) $\sum_{n=0}^{\infty} 2^n x^n = 1 + 2x + 4x^2 + 8x^3 + \cdots + 2^n x^n + \cdots = \frac{1}{1-2x}$ ✓

Need $|2x| < 1$

$2x < 1$ and $2x > -1$

$x < \frac{1}{2}$ and $x > -\frac{1}{2}$

The series represents $f(x) = \frac{1}{1-2x}$ on

$\Rightarrow$ interval $\left(-\frac{1}{2}, \frac{1}{2}\right)$ ✓

(22) $\sum_{n=0}^{\infty} (-1)^n (x+1)^n = 1 - (x+1) + (x+1)^2 - (x+1)^3 + \cdots + (-1)^n (x+1)^n + \cdots$

$= \frac{1}{1+(x+1)} = \frac{1}{x+2}$ ✓ The series represents $f(x) = \frac{1}{x+2}$

Need: $|-(x+1)| < 1$

$|-x-1| < 1$

$-x-1 < 1$ and $-x-1 > -1$

$x > -2$ and $-x > 0$

$x < 0$

on

$\Rightarrow (0, -2)$

(23) $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n = 1 - \frac{1}{2}(x-3) + \frac{1}{4}(x-3)^2 - \frac{1}{8}(x-3)^3 + \cdots + \left(-\frac{1}{2}\right)^n (x-3)^n \cdots$

$= \frac{1}{1+\frac{1}{2}(x-3)} = \frac{1}{-\frac{1}{2}+\frac{1}{2}x} = \frac{2}{x-1}$

on

Need $\left|-\frac{1}{2}(x-3)\right| < 1$

$-\frac{1}{2}x + \frac{3}{2} < 1$ and $-\frac{1}{2}x + \frac{3}{2} > -1$

$-\frac{1}{2}x < -\frac{1}{2}$ $\quad -\frac{1}{2}x > -\frac{5}{2}$

$x > 1$ $\quad x < 5$

$\Rightarrow (1, 5)$

(24) $\sum_{n=0}^{\infty} 3\left(\frac{x-1}{2}\right)^n = 3 + 3\left(\frac{x-1}{2}\right) + 3\left(\frac{x-1}{2}\right)^2 + 3\left(\frac{x-1}{2}\right)^3 + \cdots + 3\left(\frac{x-1}{2}\right)^n$

$$\frac{3}{1-\left(\frac{x-1}{2}\right)} = \frac{6}{2-(x-1)} = \frac{6}{3-x}$$

series represents $f(x) = \frac{6}{3-x}$

$\left|\frac{1}{2}(x-1)\right| < 1$

$\frac{1}{2}x - \frac{1}{2} < 1$ and $\frac{1}{2}x - \frac{1}{2} > -1$

$\frac{1}{2}x < \frac{3}{2}$ and $\frac{1}{2}x > -\frac{1}{2}$ $\Rightarrow$ on $(-1, 3)$

$x < 3$ and $x > -1$

(25) $\sum_{n=0}^{\infty} \sin^n x = \sum_{n=0}^{\infty} (\sin x)^n$

$= 1 + \sin x + \sin^2 x + \sin^3 x + \cdots + \sin^n x + \cdots$

$= \frac{1}{1-\sin x}$ series represents $f(x) = \frac{1}{1-\sin x}$

need $|\sin x| < 1$

$\sin x < 1$ and $\sin x > -1$

$-1 < \sin x < 1$

everywhere except odd multiples of $\frac{\pi}{2}$ i.e $\{x \mid x = (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}\}$

(27) $\frac{d}{dx}\left(\frac{1}{1-2x}\right) = \frac{d}{dx}\left(1 + 2x + 4x^2 + 8x^3 + \cdots + 2^n x^n + \cdots\right)$

$-(1-2x)^{-2}(-2) =$

$\frac{2}{(2x-1)^2} = 2 + 8x + 24x^2 + \cdots 2n\,x^{n-1}$

$= \sum_{n=1}^{\infty} 2^n \cdot n\, x^{n-1}, \quad -\frac{1}{2} < x < \frac{1}{2}$

(28) $\frac{d}{dx}\left(\frac{1}{x+2}\right) = \frac{d}{dx}\left(1 - (x+1) + (x+1)^2 - (x+1)^3 + \cdots + (-1)^n (x+1)^n + \cdots\right)$

$-\frac{1}{(x+2)^2} = -1 + 2(x+1) - 3(x+1)^2 + \cdots (-1)^n \cdot n (x+1)^{n-1} + \cdots$

(31) $\int_0^x \frac{1}{1-2t}\,dt = \int_0^x \left(1 + 2x + 4x^2 + 8x^3 + \cdots + 2^n x^n + \cdots\right)$

$-\frac{1}{2}\ln|1-2x| = x + x^2 + \frac{4}{3}x^3 + 2x^4 + \cdots$